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13. ABSTRACT ***(Maximum 200 Words)***

This three-year program of research concerned the development of new techniques for diagnosing, modeling and controlling nonlinear systems. The primary focus was on low-dimensional systems and the detection of bifurcation precursors in noisy environments. The major objective of the research was to extend our previous work on symbolic time series analysis to weakly non-stationary systems which are undergoing a qualitative change in their behavior. The aim is to develop a general, practical and robust detection scheme to provide early warning of impending instabilities.

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MODELING AND DIAGNOSING NONLINEAR SYSTEMS

FINAL REPORT FOR AFOSR GRANT NUMBER F49620-98-1-0144

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Executive summary

Statement of objectives

This three-year program of research concerned the development of new techniques for diagnosing, modeling and controlling nonlinear systems. The primary focus was on low-dimensional systems and the detection of bifurcation precursors in noisy environments. The major objective of the research was to extend our previous work on symbolic time series analysis to weakly non-stationary systems which are undergoing a qualitative change in their behavior. The aim is to develop a general, practical and robust detection scheme to provide early warning of impending instabilities. To aid in the accomplishment of this objective, we have developed an in-house collaboration with the W&M AMO group. This collaborative effort grounds the new techniques in a real-world problem: the active control of instabilities in ultrafast pulsed lasers, a challenging environment of great technological importance.

Methods employed

The symbol statistics of a dynamical system is computed by discretizing the analog output from the systems of interest (if the output is continuous or high resolution digitized data), or by using the discrete data directly (if the sensors are of very low dynamical resolution). Detecting transitions in noisy dynamical behavior is then reduced to detecting changes in the symbol statistics. This has been demonstrated to work on numerically generated signals.

Significance of results

Symbolic time series methods are robust to noise and run fast in real-time. Such methods have successfully been applied to the early detection of instabilities in V-8 engines and incorporated into single-bit control schemes. Symbolic precursor methods are essential in situations where the time urgency of the problem precludes methods which are more CPU intensive.

Objectives

The goal of this three year research program was to improve the diagnosis, modeling, and control of nonlinear systems, particularly in noisy environments. The major objectives were: 1] To extend our previous work on symbolic time series analysis to weakly non-stationary systems, with special emphasis on the detection of instability precursors and to incorporate these new methods into real-world active control schemes. 2] To develop improved input/output modeling of nonlinear devices.

Brief summary of new results

Symbolic bifurcation precursors. Our goal was to develop detectors which provide early warning of noise-driven instabilities. Using numerically generated data, we showed that it is possible to detect changes in the character of noise-driven fluctuations prior to transcritical bifurcations and sub-critical Hopf bifurcations. Precursor detection is possible using sliding windows with only ~ 20 data points. Applying this method to the CPM laser instability problem (to be discussed momentarily) led to the realization that previous modeling of CPM instabilities was inadequate to guide the search for precursors. Significant effort was expended on this modeling effort, which we deemed essential on scientific grounds. Our theoretical model suggests that the CPM laser exhibits two basic instabilities: a pitchfork and a saddle-node. We have also developed a symbolic variant of a recently proposed method for detecting 'information flow' between dynamical subsystem. This will also be of use in the search for instability precursors.

Improved input/output modeling of nonlinear devices (Performed by Dr. Reggie Brown, in collaboration with Hewlett-Packard. Dr. Brown was co-PI on the grant for the first two years.). In the design of complex circuits one wants to know what output will occur from a given sequence of inputs. At present, Hewlett-Packard uses specialized machines that do this. One connects the component to the test-measurement device, which then sends inputs and measures outputs. From this I/O data the test-measurement device constructs a model for the component. The long-term research program (in collaboration with Dr. N. Tuffilaro of HP) concerned applying Dr. Brown's previously developed modeling techniques to this problem and to obviate the need for expensive specialized machinery.

Background discussion: *Symbolic bifurcation precursors*

Symbolic methods are attractive for applications where the detection/classification task is time-urgent and one has only a few low-resolution sensors. In symbolic time series analysis, one converts the measured signal to a binary string via some transduction rule (for example, by passing it through a threshold function). The *symbol statistics* are then estimated by choosing a window of length N and observing the frequencies of all symbol sub-patterns up to some length $L << N$.

We focus on weakly non-stationary systems where slow parameter variation leads to a bifurcation. Of particular interest are noise-driven sub-critical bifurcations. These are the most dangerous in practice. For concreteness, consider the $1D$ system:

$$\dot{x} = -x(x-\mu)(x-1) + \eta(t). \tag{1}$$

Here $\eta(t)$ is the noise driving (assumed gaussian-white for simplicity). For $\eta = 0$ there are three fixed points: $x = (0, \mu, 1)$. For $0 < \mu < 1$ the fixed point at μ is unstable, while the other two are stable. When μ passes through zero the deterministic system ($\eta = 0$) undergoes a transcritical bifurcation. We start the system with initial condition $x(0) = 0$, set $\mu_0 = .25$ and keep the noise amplitude small $< \eta^2 >\ll \mu_0$. As t increases we slowly decrease $\mu \downarrow 0$. The noise driving causes the system to transition to the other stable fixed point (at $x = 1$) prior to the deterministic bifurcation.

Given a sample of the behavior of the system away from the bifurcation, we choose a sampling interval Δt which is long compared to the stationary decorrelation time and convert the time series into a binary string. Taking sliding windows of length N we count the instances of the pairs 00, 01, 10 and 11 and compute the 'small-sample Shannon entropy':

$$\mathcal{E} \equiv \frac{-1}{log4} \sum_j \frac{n_j}{N} log \frac{n_j}{N}, \tag{2}$$

where $j = 00, 01, 10$ and 11. Experience shows that this statistic is sensitive to changes in the character of the fluctuations. Even using short windows (for example, $N = 17$) we can detect the growing 'order' of the fluctuations as the bifurcation is approached. By examining what occurs in particularly significant entropy dips, we can detect which bifurcation is occurring, something which is hard to do if the window size is chosen too large.

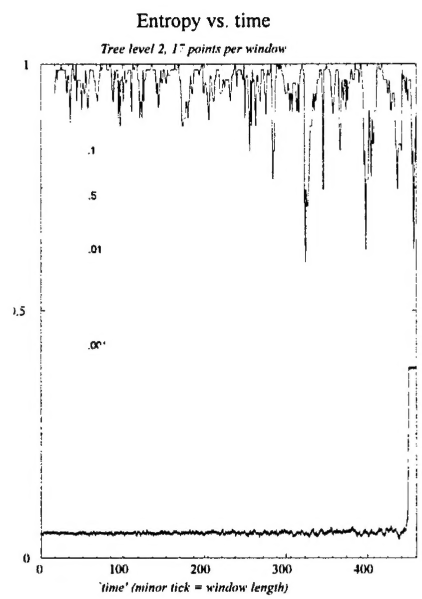

Figure 1:

In Figure 1, we plot the time series for Eq. (1), as well as the position of the unstable fixed point, $\mu(t)$, and the entropy $\mathcal{E}(t)$. The signal $x(t)$ is shown at the bottom. The signal has been given a slight positive shift and rescaled in amplitude for ease of viewing. (The

vertical scale refers to the entropy, which ranges from zero to one.) The 'step' at very late times in $x(t)$ is the transition to the new equilibrium point $x = 1$. The smooth curved line which intersects $x(t)$ is the unstable fixed point $\mu(t)$. The curve at the top of the figure is $\mathcal{E}(t)$, computed using sliding windows of 17 data points. The window length is indicated by the small tics along the x-axis. The four horizontal lines indicate the probability of observing various low entropy values given the null hypothesis that the 17-symbol sequence was generated by a Bernoulli (coin flip) process. The lines indicate probabilities of .1, .05, .01 and .001. The later dips in the entropy are clearly significant.

During this project, we also began collaborative work with the laser optics group at W&M, led by Prof. Bill Cooke. This group has a colliding pulse mode-locked (CPM) laser which can generate ultrashort pulses of $\sim 100fs = 10^{-13}s$ pulse-width (Figure 2 provides a schematic of the apparatus). The laser exhibits a 'drop-out' instability where the mode-locking is lost. Our goal is to improve the stability of the laser and achieve shorter pulse-widths. As a nonlinear control problem, the CPM laser is particularly challenging because the dynamics is extremely stiff and close to neutral stability over the entire operating range (as will be shown). Therefore, it is very susceptible to noise effects. This research project has involved a significant modeling effort in order to uncover the nature of the instabilities associated with the loss of mode-locking (and resultant loss of stable pulse behavior). At present, we have: 1] carried out a careful bifurcation analysis of a simple nonlinear model which captures many of the qualitative features of the CPM dynamics, 2] developed a scaling law for noise-driven escape from the mode-locked state which can be tested experimentally, 3] begun preliminary studies of a symbolic variant of Schreiber's recently proposed *transfer entropy* (for detecting changes in the directional couplings between subsystems).

1] Bifurcation analysis of CPM laser dynamics: We include the leading order effects of frequency-dependent gain and loss, dispersion, and nonlinearity. We adopt the approach of keeping the model as simple as possible until it is clear something essential is missing, because this improves the prospect of arriving at a simple control law. Because the pulse width Δt is much smaller than the round-trip time ($\Delta\tau \sim 10^{-8}\ s$), one can develop a dynamical model which describes how the pulse shape changes from one round-trip to the next (i.e. a *Haus*-type model),

$$\psi^{(n+1)}(t) = \left[\alpha(|\psi^{(n)}(t)|^2) - \beta\frac{d^2}{dt^2}\right]\psi^{(n)}(t). \tag{3}$$

This function map already assumes that the pulses are mode-locked and that they collide in the absorber, hence this is a map 'on the synchronization manifold'. Once the bifurcation behavior of this map is understood, extension of the model off the synchronization manifold (which requires a higher-dimensional model) will be pursued. The function $\psi^{(n)}(t)$ is the pulse *envelope* on the n^{th} pass through the system. The second-order derivative term describes the (leading order) effects of frequency-dependent gain and dispersion. The factor $\alpha = \alpha(|\psi^{(n)}|^2)$ is the net gain per-pass of the pulse, including nonlinear saturation of gain and absorption. Measurements by the W&M laser optics group suggest that stable pulse behavior occurs

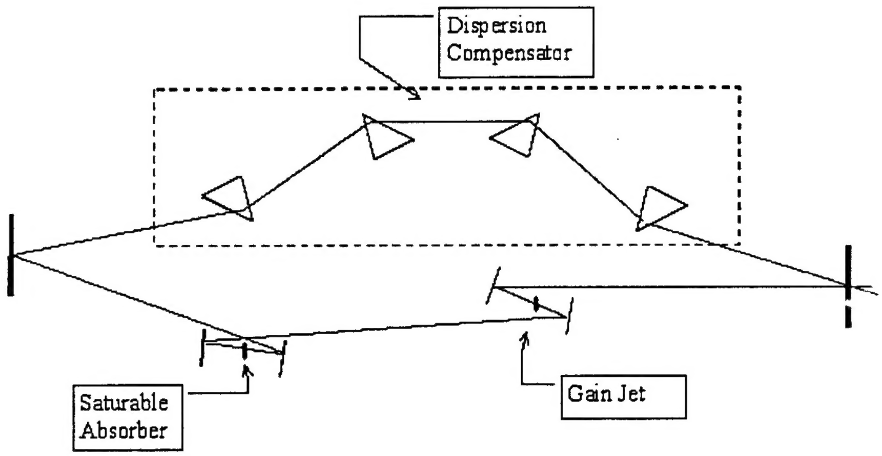

Figure 2: Simplified diagram of the colliding pulse mode locked (CPM) laser. The laser has two dye jets (one is the gain jet, the other is a saturable absorber). The gain jet is pumped by a second laser (not shown). The round trip length for pulses is about 10 meters, leading to a pulse repetition rate of about 300 MHz.

only when the loss is fully saturated, hence a small-amplitude expansion of $\alpha(|\psi|^2)$ (which is usually adopted) is suspect. Therefore, we examine a fully saturable model, but first examine bifurcations without the dispersive term.

The transfer map, T(z): Passage through a gain medium which is pumped to create an inverted population of two-level atoms leads to the standard *laser map*, which we write in the form

$$z^{out} = G(z^{in}) = R\left(1 + \frac{g}{1 + a|z^{in}|^2}\right) z^{in}, \tag{4}$$

while passage through a saturable absorber can be modeled via

$$z^{out} = L(z^{in}) = \left(1 - \frac{l}{1 + b|z^{in}|^2}\right) z^{in}. \tag{5}$$

Here, $z = Ae^{i\theta}$ is the amplitude and phase of the pulse, R accounts for cavity losses (e.g. loss on mirrors), g is the gain per-pass at threshold (A=0), l is the loss per-pass at threshold, and a and b are nonlinear saturation parameters. These two maps are *composed* to generate the *transfer* or *CPM map*, $T(z) \equiv \alpha(|z|^2)z$: $T(z) \equiv G(L(z)) = (G \circ L)(z)$.

The fixed point equation $z_* = T(z_*)$ contains five parameters (R, g, l, a, b), of which (g, l, a, b) are candidates for control knobs. Of prime importance is to note the fact that over the entire range of operating parameters, this map is extremely stiff and deviates from the identity by only $T(z) - z \sim 10^{-3} - 10^{-4}$. This map is rational in z, and the fixed point equation is solvable by analytical methods. This is of great importance, since the stiffness of the equation makes numerical root finding difficult.

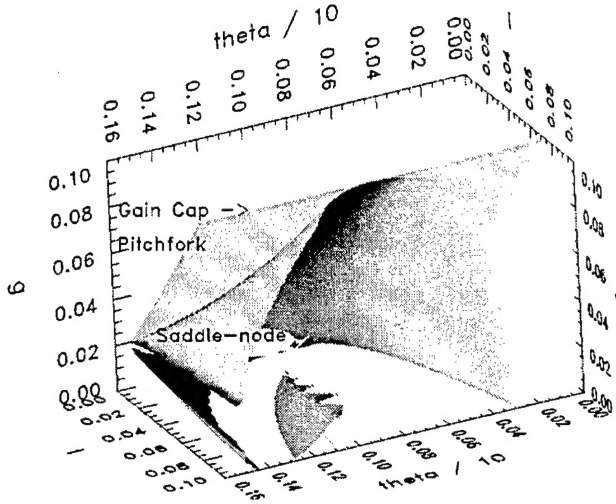

Figure 3: The critical surfaces for the colliding pulse map. Pulses only occur in the lower left part of the diagram. Increasing the gain (moving upward on the diagram) leads to a pitchfork bifurcation. Changing the ratio of the nonlinear saturation parameters in the gain and absorber (denoted 'theta' in the figure) leads to a saddle-node bifurcation.

When all parameters are real the phase dynamics is trivial and the amplitude map exhibits only two bifurcations of physical importance. 1] There is a pitchfork bifurcation at $A = 0$. This pitchfork bifurcation corresponds to the loss of pulses and the onset of CW behavior. In fact, prior to true CW operation the system hops into a 'long pulse' mode, suggesting that the pulse shape dynamics is important for the pitchfork bifurcation. 2] There is a saddle-node bifurcation at finite amplitude where the pulse state 'drops out' catastrophically and the system relaxes to $A = 0$. (We note in passing that, to our knowledge, previous CPM models have missed the possibility of the saddle-node because of the adoption of a small-amplitude expansion for the nonlinear effects.) Effective potential arguments using Eq. (3) strongly suggest that pulses exist only when the map is stable at threshold ($A = 0$).

A W&M graduate student, Mr. George Andrews, has performed a careful bifurcation analysis of the CPM map. The results are summarized in Fig. (3). The axes are the gain g (vertical), the loss l and the function $\theta \equiv tan^{-1}(b/a)$. The sloping planar surface is the *gain cap.* Above the gain cap, the origin is unstable at threshold and the system operates in CW mode. Below the gain cap, and inside the curved critical surface or to its right, there are no positive real roots. Hence, the system relaxes to $A = 0$. For parameters below the gain cap and to the left of the curved critical surface, the origin is stable and there is a single root at finite amplitude. This is the pulse formation region. We note that these two critical surfaces are tangent for some parameter values, suggesting the presence of bifurcations of co-dimension higher than 1. The physical importance of this is unclear and is currently

under investigation.

2] Noise driven bifurcations in the CPM laser: We can now ask how these bifurcations will be affected by noise in the laser cavity. The goal is to extract scaling laws for the average escape time from the stable fixed point as a function of parameters in the vicinity of the bifurcations. Experimental verification of these scaling laws would provide strong support for the present modeling approach. The examination of noise effects will also aid in the identification of noisy bifurcation precursors, which is the primary goal of the project.

We first examine additive noise which is uncorrelated from one round trip to the next, due to spontaneous emission or other fast processes. The analysis of the saddle-node dropout is carried out on the discrete map. (As already mentioned, the pitchfork bifurcation appears to involve non-trivial pulse shape dynamics and, hence, requires the use of the differential-delay equation. This problem will be examined as soon as the simpler saddle-node study is complete.) The transfer map with noise becomes

$$z^{(n+1)} = T(z^{(n)}) + \eta^{(n)} \tag{6}$$

where η is assumed to be a gaussian-white process $<\eta^{(n)}\eta^{(m)}> = \sigma_\eta^2\delta_{mn}$. Linearizing about the stable fixed point we arrive at a standard linear *Ornstein-Uhlenbeck* model. Using this model, we can estimate how the average escape time depends upon the parameters. This calculation has been done and will soon be compared with a direct numerical simulation using the fully nonlinear CPM map (6) to test the validity of the approximations used. The scaling law will, of course, also be tested experimentally.

3] The search for symbolic bifurcation precursors of CPM laser instabilities: Prior to our modeling efforts, our early symbolic studies of time series taken from the CPM laser were inconclusive. This led us to develop the CPM model and to our present understanding of what types of bifurcations to expect. We are now in a better position to uncover a precursor. At this point it is fairly clear that such a precursor will probably involve information about the cross-correlations between the colliding pulses and the pump laser, which is their ultimate source of energy. Schreiber has recently proposed a statistic called the 'transfer entropy' which purports to measure the *directional* flow of information between subsystems (the standard mutual information cannot do this since it is defined symmetrically with respect to exchange of the subsystems). Schreiber's original work involves the calculation of correlation exponents, which is CPU- and data-intensive and sensitive to noise effects. With Mr. Michael Ricci, a summer NSF REU student, we have examined a symbolic variant of Schreiber's transfer entropy and found that it is possible to detect the directionality of coupling between two coupled maps. This symbolic version runs very fast and requires using only a few thousand points to detect the directionality at a high confidence level even when the coupling is only $\sim 5\%$ of the signal strength. We will pursue this approach further, and use it as a tool to search for bifurcation precursors in time series generated from the CPM models as well as data taken from the CPM laser.

Significant Collaborators and Personnel Supported by this Grant:

The CPM laser work is being carried out in collaboration with the W&M laser optics group, led by Prof. W. E. Cooke and with the support of graduate student, Wei Zhang. The symbolic variant of Schreiber's transfer entropy has been developed with Mr. M. Ricci, a summer NSF REU student. We also have an ongoing collaboration with Mr. D. Weaver (Lt. Col., USAF Ret.) of St. Leo's University, who will follow-up on Mr. Ricci's preliminary results.

Senior Personnel Supported:
E. R. Tracy (1 month/year for 3 years).
Reggie Brown (1 month/year for 2 years).

Post-doctoral fellows:
Radu Manuca (post-doc), was supported full-time for 1 year (June 1998-June 1999).

Graduate Students Supported:
G. A. Andrews (fall 1999-fall 2000).
W. Zhang (summer 2000-fall 2000)

Other personnel:
Dennis Weaver (Lt. Col. USAF Ret., 8 months @ 1 day/week)

Relevant publications during project period:

1. "Synchronization of chaotic systems: stability of chaotic sets in invariant manifolds", R. Brown and N. F. Rulkov, CHAOS **7** (1998) 395.

2. "Scaling relations for non-normal transitions", E. R. Tracy and X.-Z. Tang, Phys. Lett. A, **242** (1998) 239.

3. "Data compression and information retrieval via symbolization", X.-Z. Tang and E. R. Tracy, CHAOS **8** (1998) 688.

4. "Takens-Bogdanov random walks", E. R. Tracy, X.-Z. Tang and C. Kulp, Phys. Rev. E, **57** (1998) 3749.

5. "Approximating the mapping between systems exhibiting generalized synchronization"; R. Brown, *Phys. Rev. Letts.* **82** (1998) 4835.

6. "Constructing transportable behavioral models for nonlinear electronic devices"; D. M. Walker, R. Brown, and N. B. Tufillaro *Phys. Letts.* **255A** (1999) 236.

Interactions/Transitions:

Presentations at meetings and/or seminars:

1. E. R. Tracy, *Introduction to symbolic time series analysis*, seminar presentation, Plasma Theory Group, Lawrence Berkeley National Laboratory, January 1998.
2. E. R. Tracy *Introduction to symbolic time series analysis,* seminar presentation, Dynamics Group, United Technologies Research Center, January 1998.
3. Reggie Brown, *Applications of synchronization to detection and control*, seminar presentation, Dynamics Group, United Technologies Research Center, January 1998.
4. E. R. Tracy, *Introduction to symbolic time series analysis,* seminar presentation, Physics Department, GA Tech, March 1998.
5. G. A. Andrews and E. R. Tracy, *Bifurcation analysis of the Colliding Pulse Mode-locked laser*, poster presentation at Dynamics Days, January 2000.
6. G. A. Andrews, E. R. Tracy and W. E. Cooke, *Bifurcation analysis of systems exhibiting spontaneous pulse formation via passive mode-locking*, Sherwood Fusion Theory Meeting, April 2000.
7. Reggie Brown, Invited talk and visit to the Issac Newton Institute, Cambridge England, Sept. 1998.
8. Reggie Brown, Invited talk at SIAM Annual Meeting, Atlanta, GA, May, 1999.
9. Reggie Brown, Organized mini-symposium on "Applications of nonlinear dynamics to problems in industry and nature' at SIAM Conference on Applications of Dynamics, Snowbird, UT, May, 1999.
10. E. R. Tracy and D. M. Weaver, *Symbolic analysis of non-stationary time series,* American Physical Society, Centennial Meeting, Atlanta, GA, March 1999, poster.
11. E. R. Tracy and D. M. Weaver, SIAM, *Symbolic analysis of non-stationary time series,* Applications of Dynamics Meeting, Snowbird, UT, May 1999, poster.
12. G. A. Andrews and E. R. Tracy, poster presentation at SIAM Pacific Rim Dynamics Meeting, Maui, HI, August 2000.

Transitions: (Note: these have already been reported in annual reports during the project period)

1. Oak Ridge National Lab/Ford Motor Company:
Work performed as part of a Cooperative Research and Development Agreement (CRADA), number ORNL-95-0337 titled "Engine Control Improvement Through Application of Chaotic Time Series Analysis".
Performer:
Dr. C. Stuart Daw
Oak Ridge National Laboratory
P.O. Box 2009
Oak Ridge, TN 37831-8088
Telephone: (615)574-0373.
Customer:
Ford Motor Co.
Dearborn, MI
Contact:
Dr. John Hoard
(313)594-1316.

Anticipated result: Using symbol statistics to do parameter fitting and control for internal combustion engines. (Aspects of the work are subject to a patent disclosure.)
Application: Improved feedback control for internal combustion engines to reduce NO_x emission and increase fuel efficiency.

2. Wm & Mary/Hewlett Packard:
Performer: Professor Reggie Brown.
e-mail: brown@predict.com.

Customer:
Hewlett-Packard Laboratories
MS4-AD, 1501 Page Mill Rd.
Palo Alto, CA 94304-1126 USA

Contact at HP:
Dr. Nicholas Tufillaro
Research Scientist
(650) 857-8696 (w).

Anticipated result: models that are more robust to changes in input signals and variations in constant driving parameters. (The latter are DC signals which are considered different from the time varying inputs.) Other anticipated results are test signals that lead to more

robust models.

Application: new test and measurement methods for components of high frequency circuits. These methods include model construction for nonlinear input output systems as well as design of test signals for these devices.

Inventions or Patent Disclosures: None.

Awards: My graduate student, Mr. George Andrews, has received a $5,000 stipend award from the VA Space Grant Consortium to support this work.